MOTIVIC INTEGRAL OF K3 SURFACES OVER A NON-ARCHIMEDEAN FIELD

VADIM VOLOGODSKY

This is a joint work with Allen J. Stewart. Let X be a smooth complete Calabi-Yau variety over a non-archimedean local field K and ω a non-zero top degree differential form on X. The motivic integral of X is an element of the ring $K_0(Var_k)_{loc}$, obtained from the Grothendieck ring $K_0(Var_k)$ of algebraic varieties over the residue field of K by inverting the element $\mathbb{Z}(-1) := [\mathbb{A}^1]$, given by the formula

$$\int_X := \sum_i [V_i^\circ](r_i - \min_i r_i),$$

where $\mathcal{V}$ is a weak Néron model of X over the ring of integers $R \subset K$, V_i° are the connected components of the special fiber of $\mathcal{V}$, and the integers r_i are defined from the equation $div\,\omega = \sum_i r_i [V_i^\circ]$. A theorem of Kontsevich, Loeser, Sebag states that $\int_X$ depends on X only and not on the choice of $\mathcal{V}$.

I will explain a formula expressing the motivic integral of a K3 surface over $\mathbb{C}((t))$ in terms of the associated limit Hodge structure.

Then, in the second part of my talk, I will construct some cohomological birational invariants of an arbitrary variety over a non-archimedean field. In particular, I will define a canonical positive pairing

$$H_d(|X^{an}_{\widehat{K}}|, \mathbb{Q}) \otimes H_d(|X^{an}_{\widehat{K}}|, \mathbb{Q}) \to \mathbb{Q},$$

where $H_d(|X^{an}_{\widehat{K}}|, \mathbb{Q})$ is the top singular homology of the Berkovich analytic space associated with X. This is a generalization of Grothendieck's monodromy pairing in the case of Abelian varieties.

Finally, I will explain a conjectural formula for the motivic integral of maximally degenerated K3 surfaces over an arbitrary non-archimedean field and if time permits prove this formula for Kummer K3 surfaces.