A symplectic version of Dubrovin's conjectures

I will tell about joint work with Vasily Golyshev and Hiroshi Iritani.

Dubrovin's ICM1998 conjectures relate quantum cohomology and derived category of coherent sheaves on the same Fano manifold. This kind of relation is somewhat surprising, however one can give a naive "proof" of the conjectures using mirror symmetry. I will tell about some generalizations of these conjectures, and recent counter-examples to these generalizations.

Finally I will formulate a version of these conjectures purely in terms of symplectic topology.

For a symplectic manifold X decomposition of the quantum cohomology algebra into a direct sum of subalgebras $QH(X) = \oplus_{i=1}^{N} R_i$ conjecturally implies the existence of a semi-orthogonal decomposition of topological K-theory into a sum of subgroups $K^\bullet(X) = \oplus_{i=1}^{N} \Lambda_i^\bullet$. Consider the subspaces $V_i = Ch(\Lambda_i^\bullet) \cup \hat{\Gamma}_X$, obtained as the images of these subgroups by Chern character map twisted by Gamma class. Then the solutions of the quantum connection with the initial conditions in the regular singular point lying inside V_i correspond to the asymptotic subspaces in the irregular singular point after the analytic continuation.