Vendo Funções

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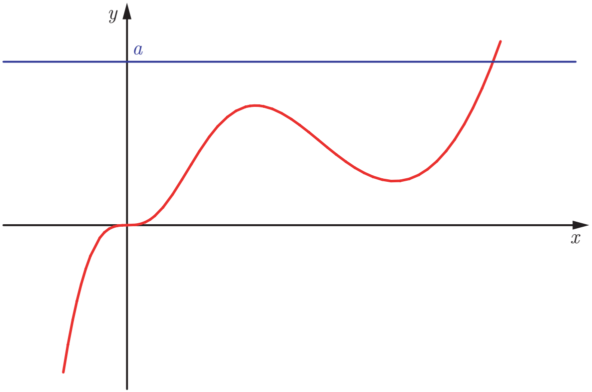
3ª Jornadas de Iniciação Científica

IMPA, Rio de Janeiro, 20 a 25 de novembro de 2006

Resolvendo equações

Quantas soluções tem $(\operatorname{arctg}(x))^3\left((x-3)^2+\frac{1}{2}\right)=a$?



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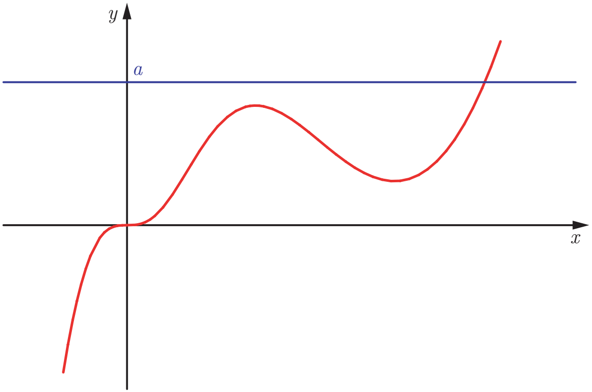

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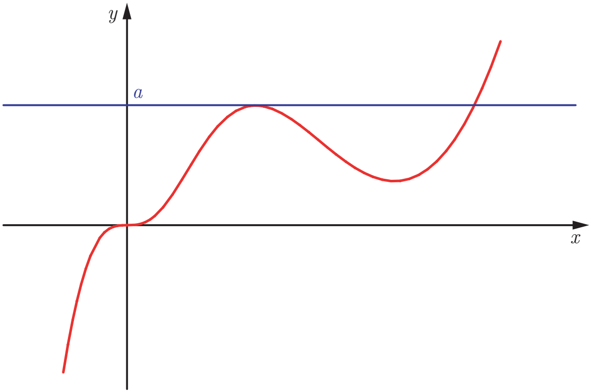

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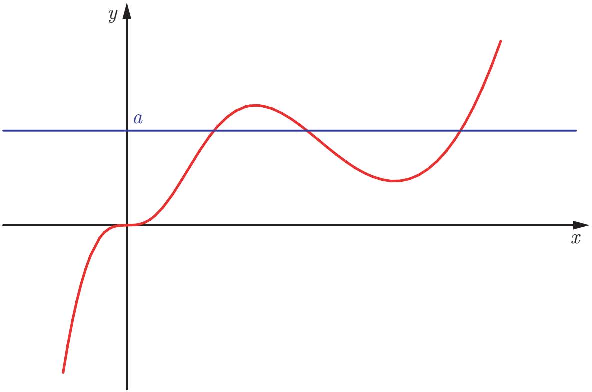

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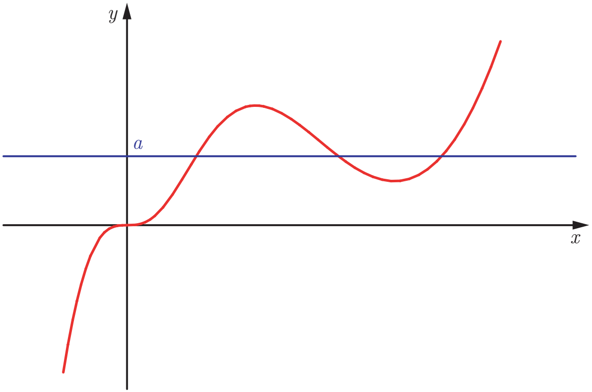

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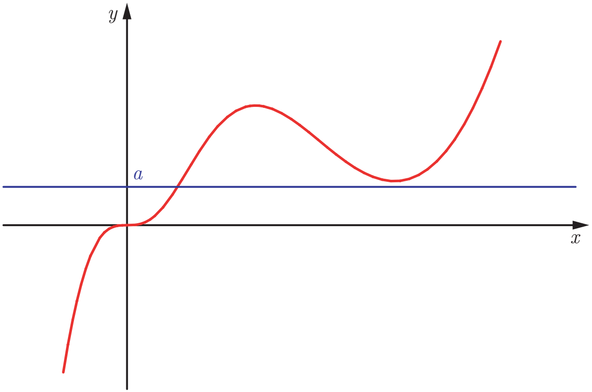

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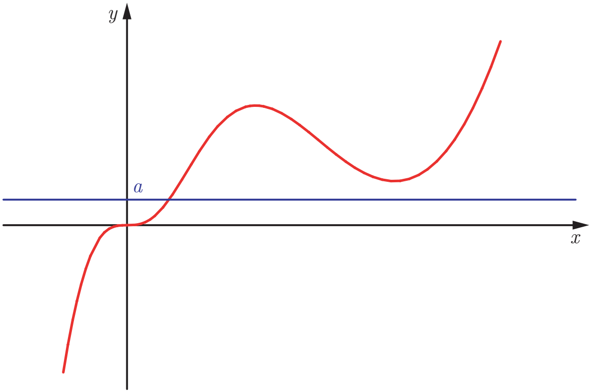

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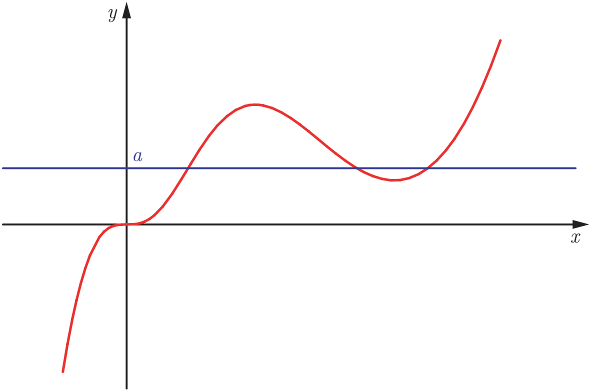

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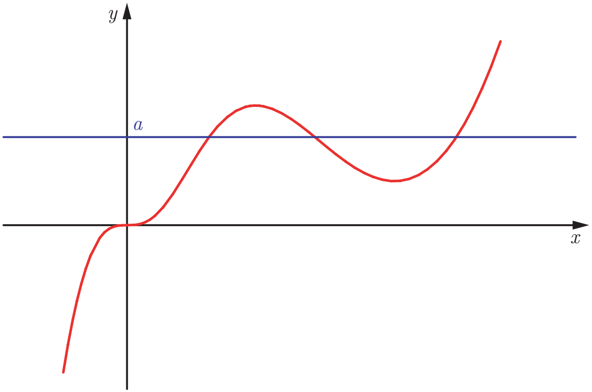

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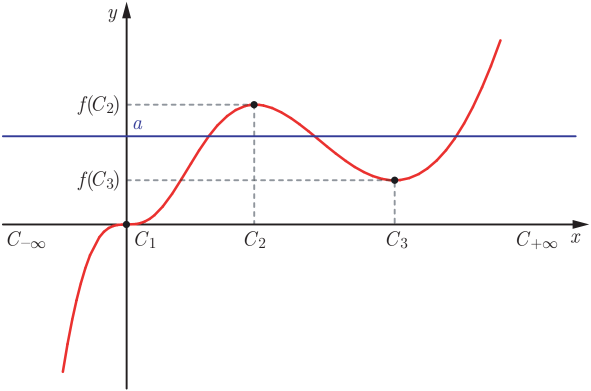

Basta saber $\{C_i\}$ e $\{f(C_i)\}$!

Resolvendo equações

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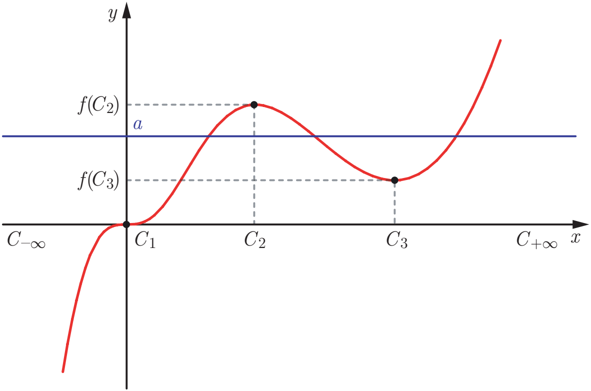
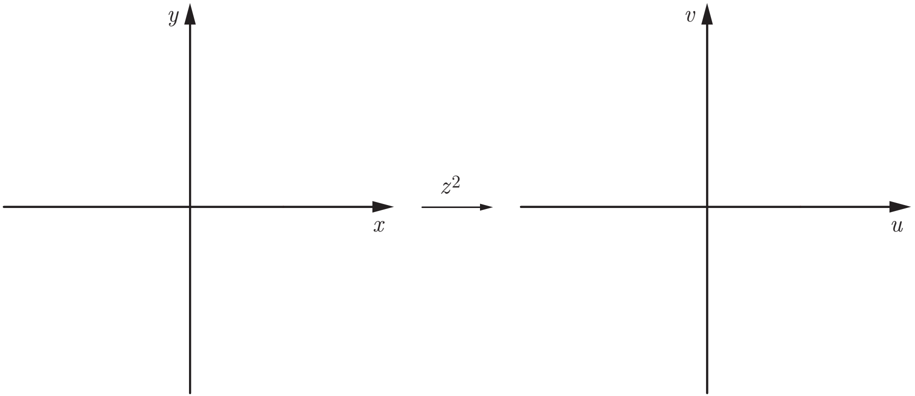


Agora, resolver não é tão difícil!

Funções analíticas: $z \mapsto z^2$



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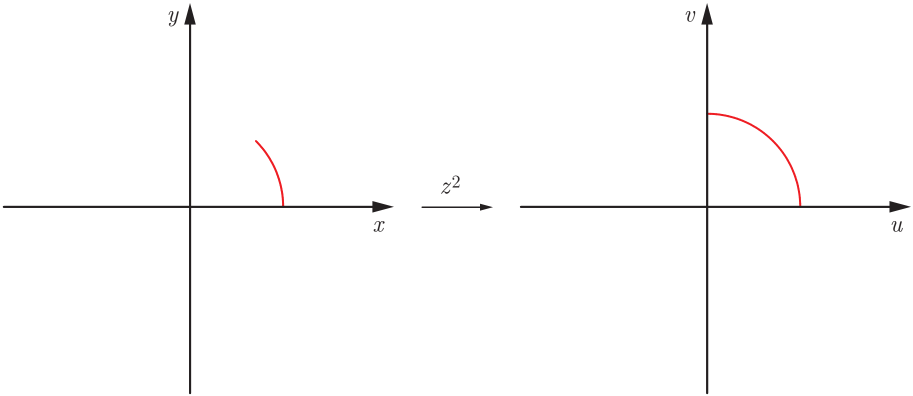

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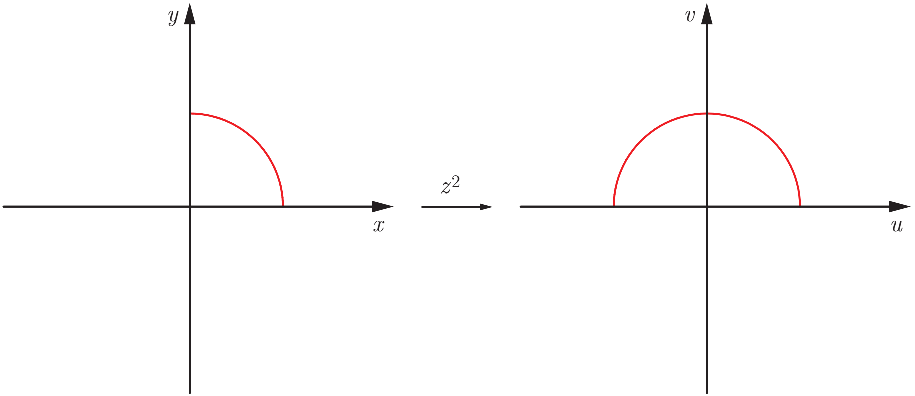

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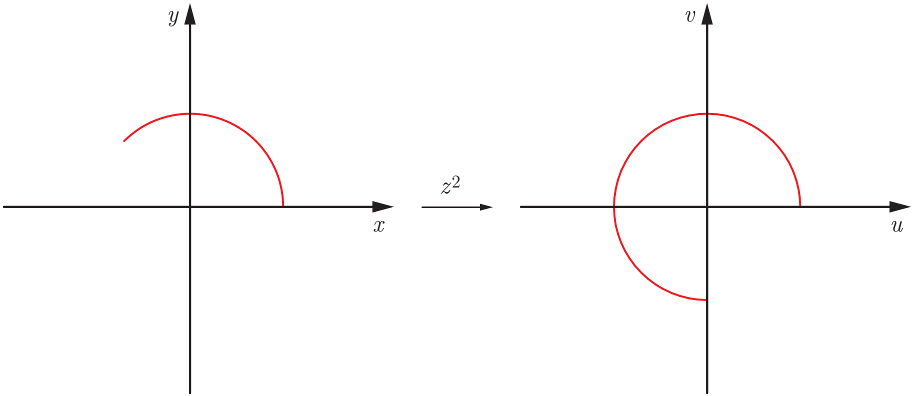

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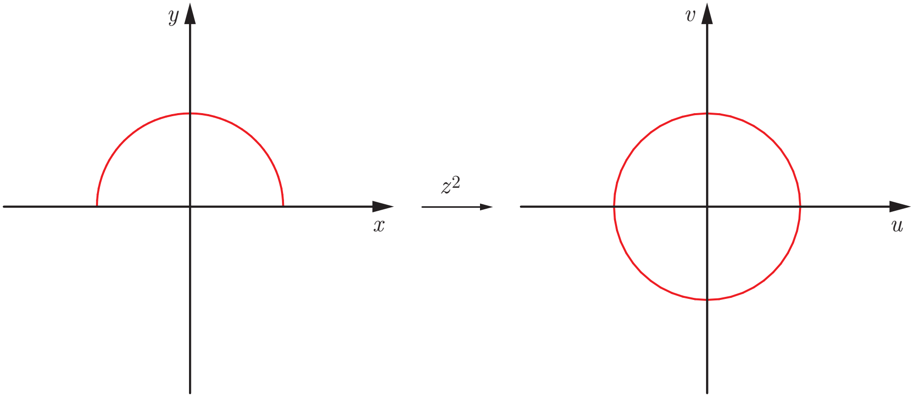

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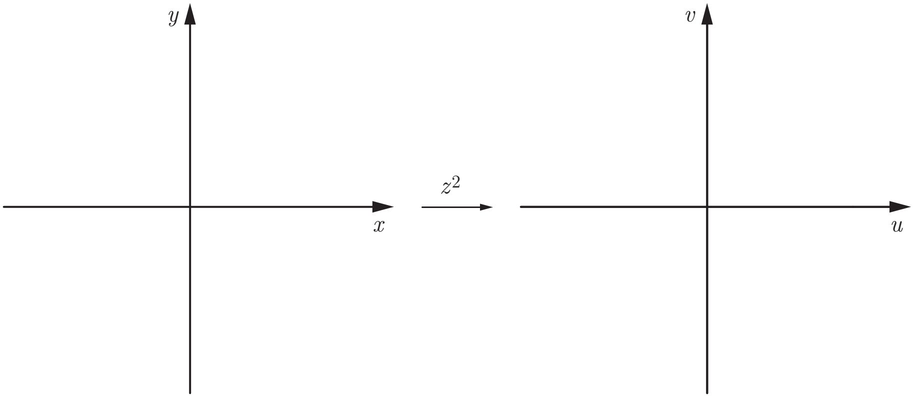

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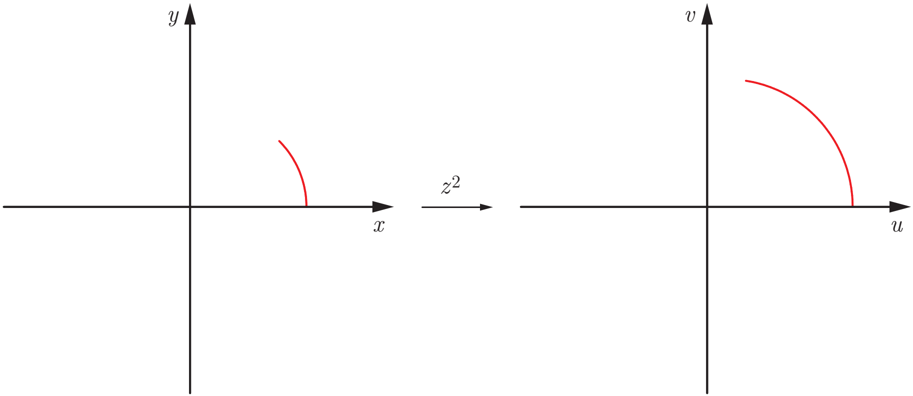

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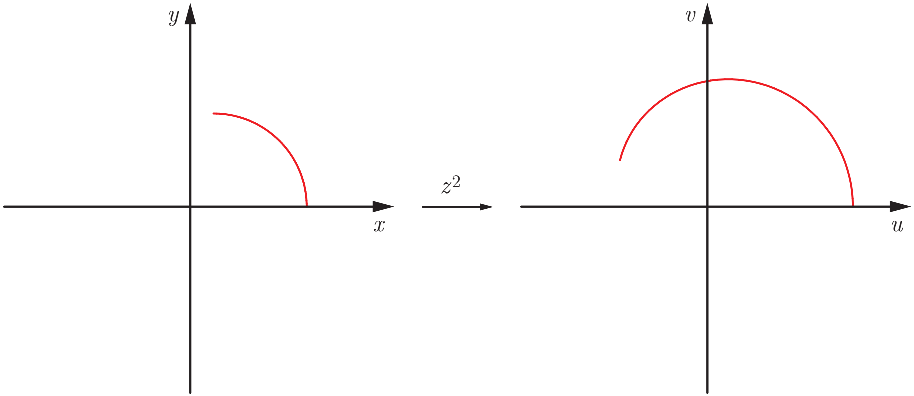

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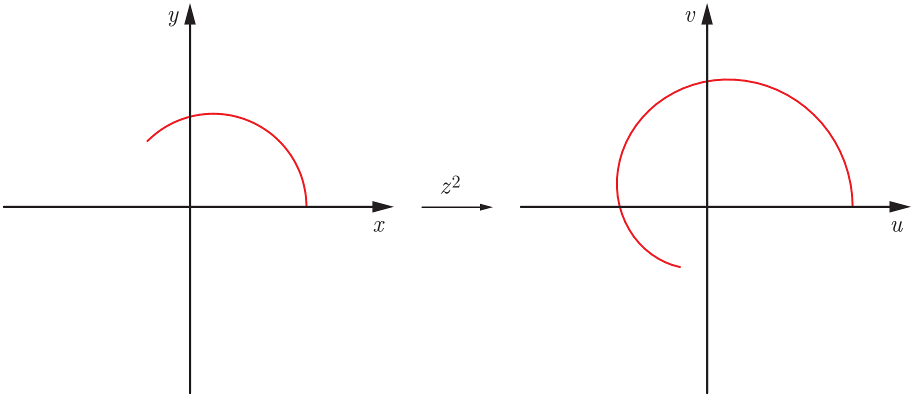

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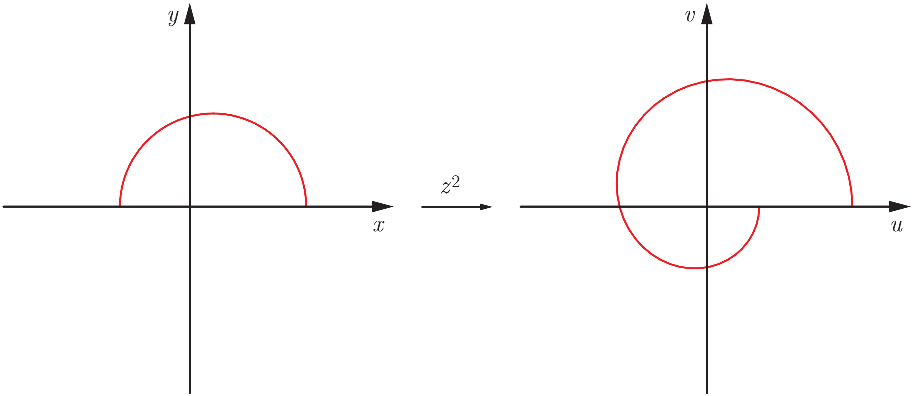

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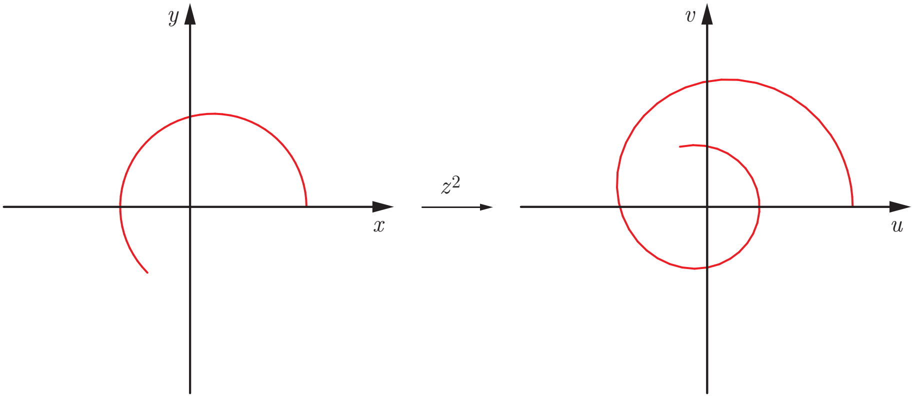

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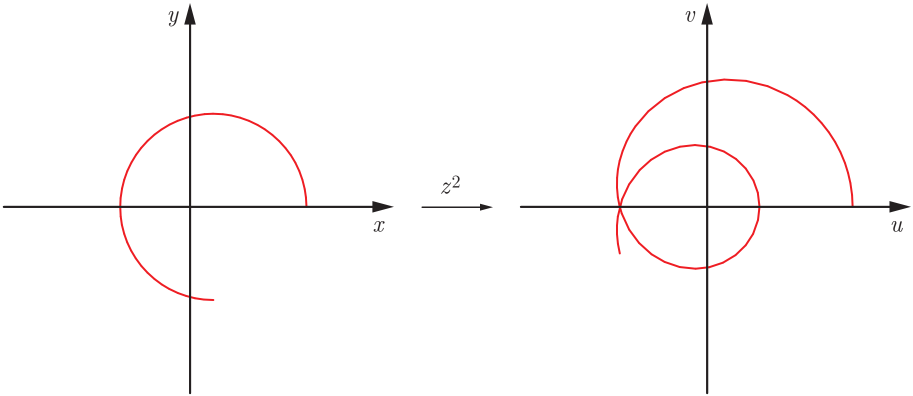

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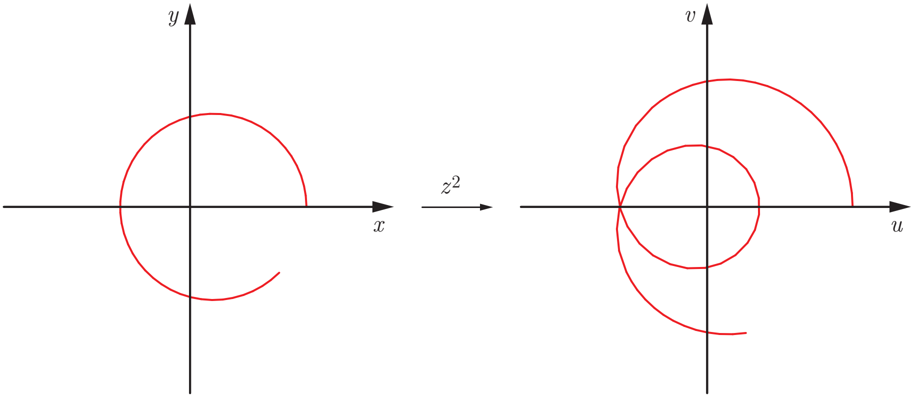

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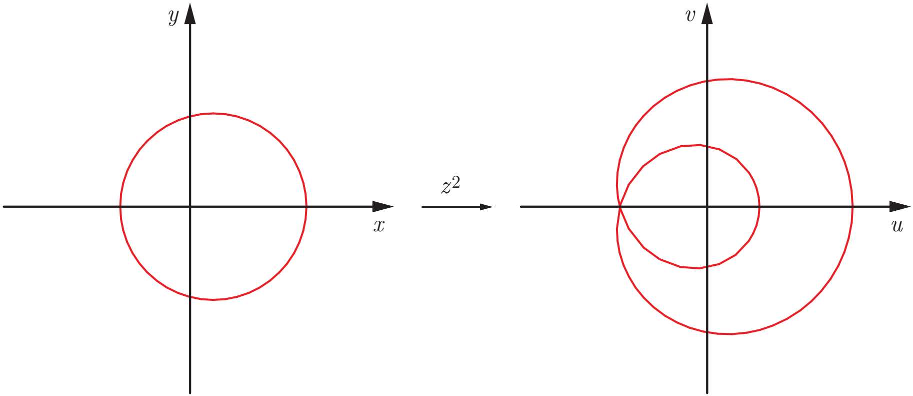
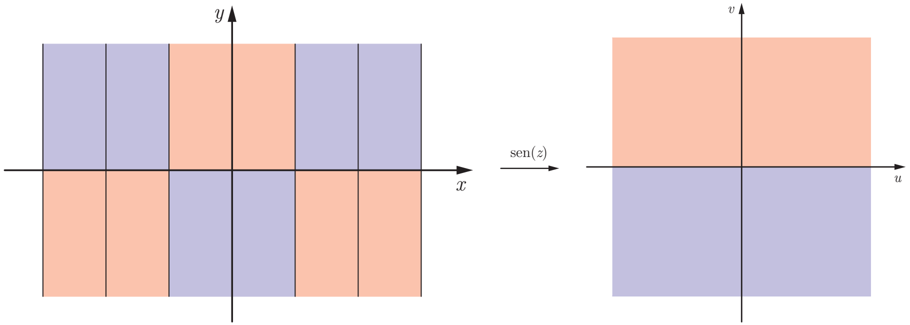


$z \mapsto \text{sen}(z)$: mapeamento conforme



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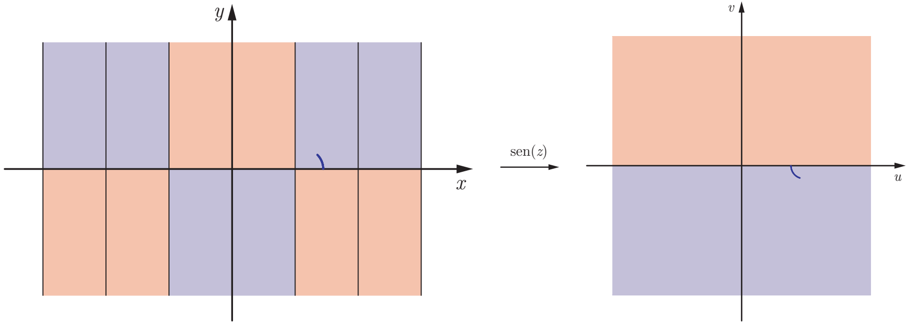

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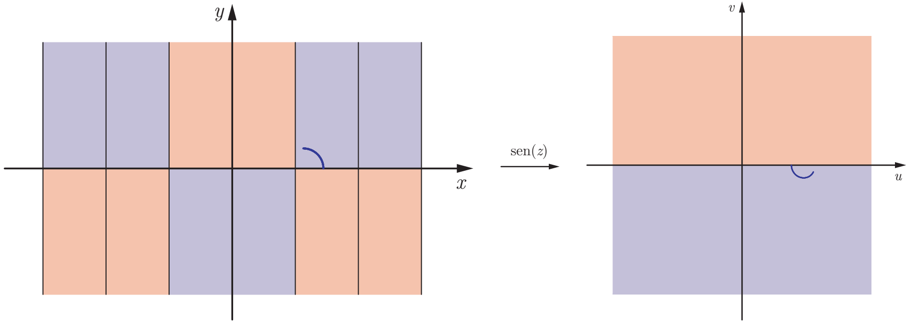

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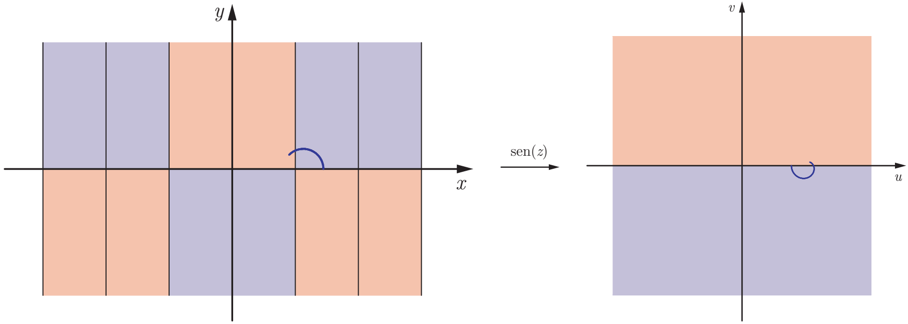

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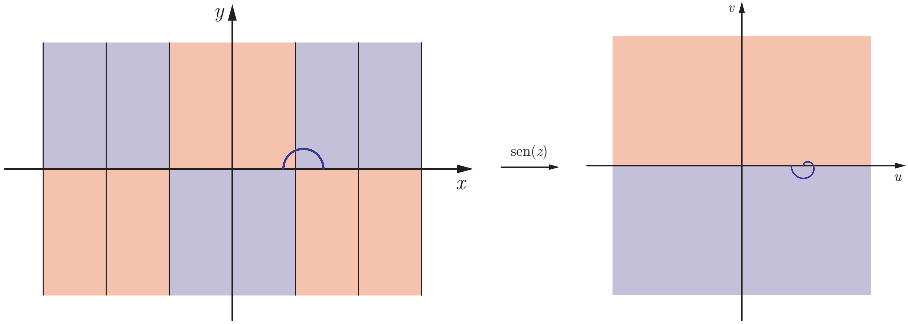

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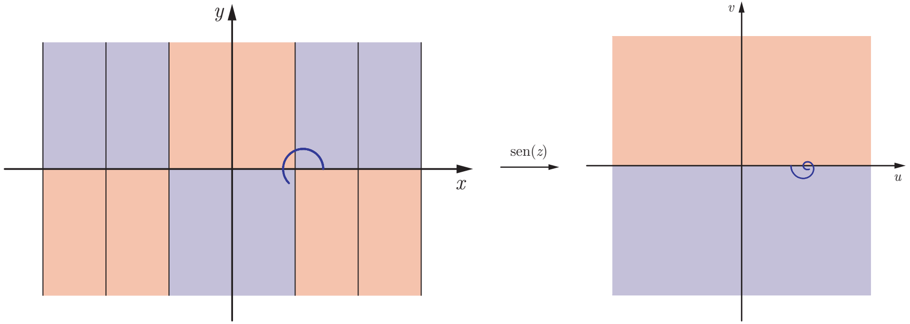

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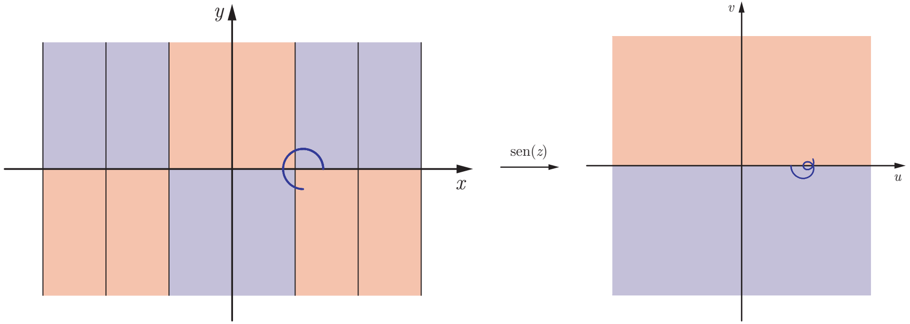

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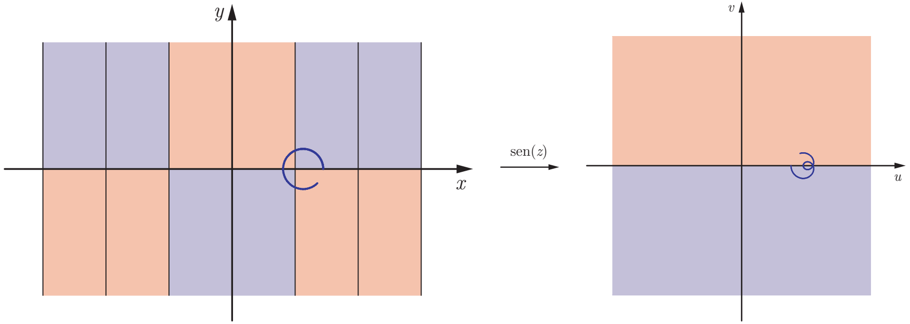

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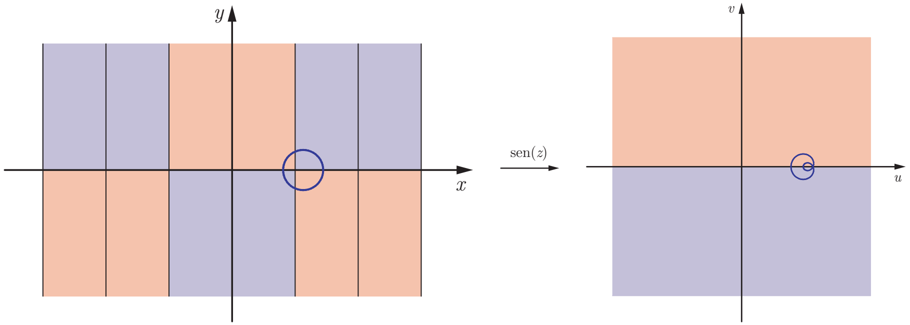

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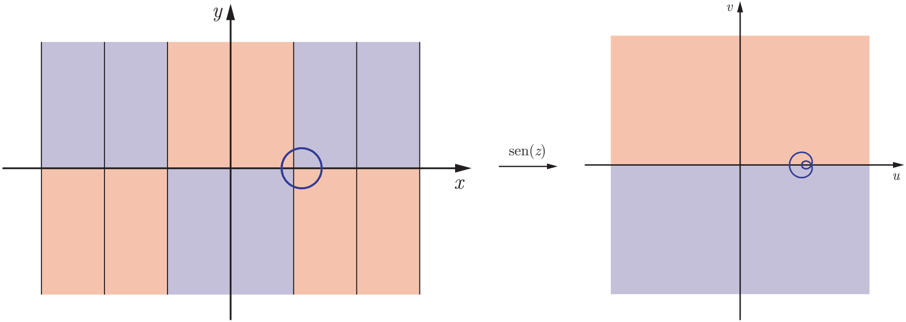

Funções analíticas preservam orientação.

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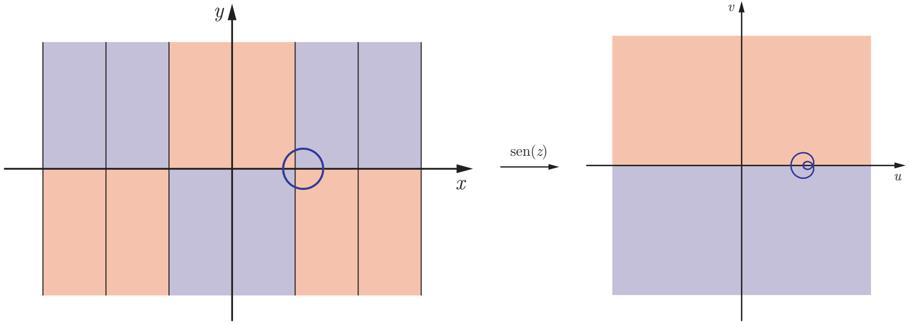

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Pontos críticos analíticos são $z \mapsto z^n$.

Uma $F\colon \mathbb{R}^2 \to \mathbb{R}^2$ suave: $z \mapsto z^2 + \overline{z}$

$$F(x,y) = (x^2 - y^2 + x, 2\,xy - y) \qquad \widetilde{F}(z) = z^2 + \overline{z}$$

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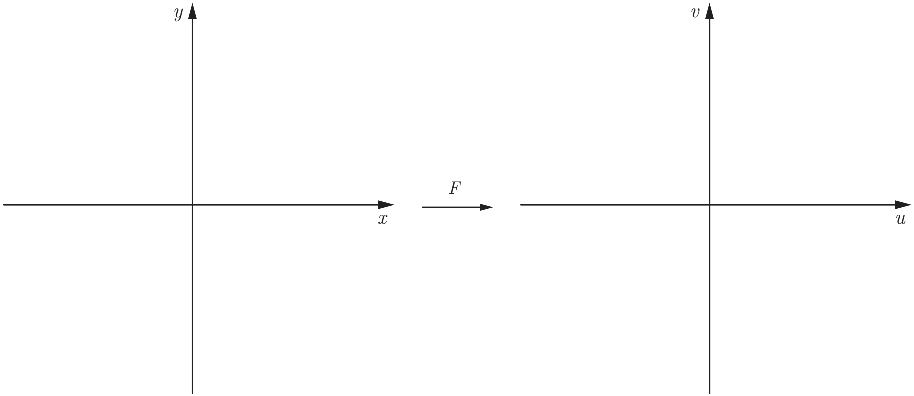

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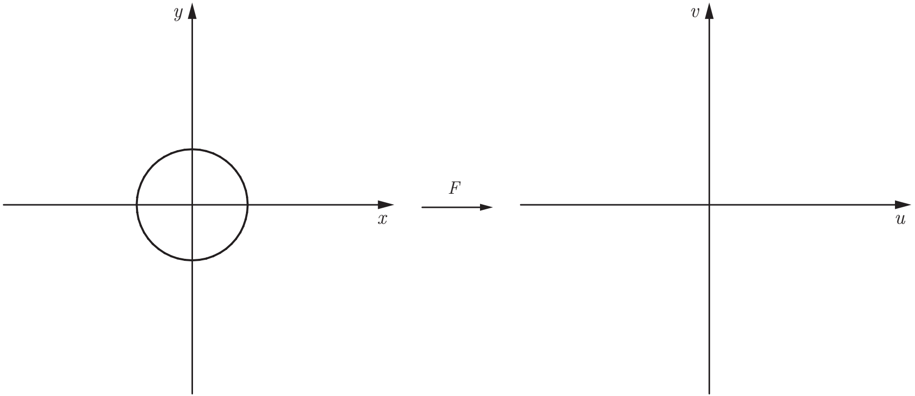

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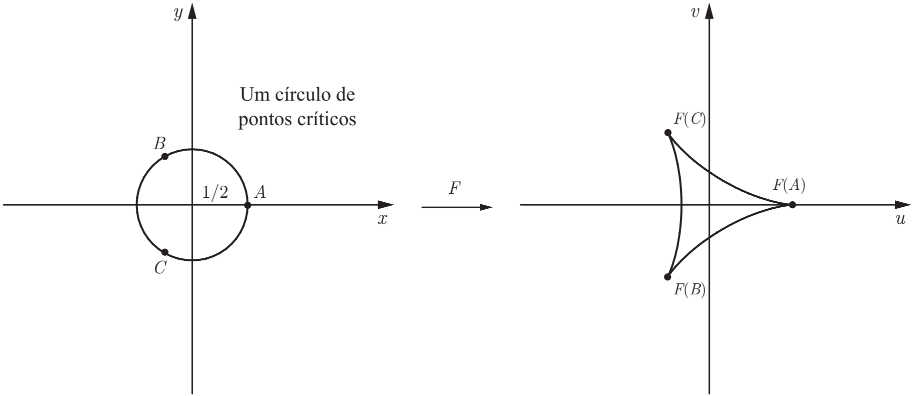

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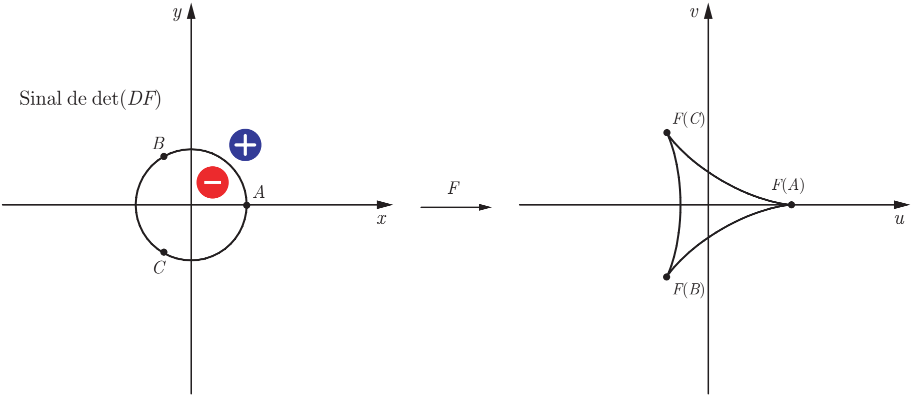

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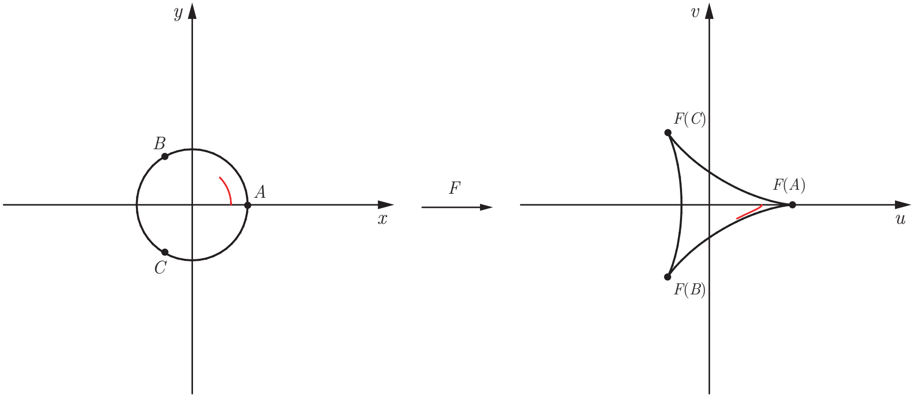

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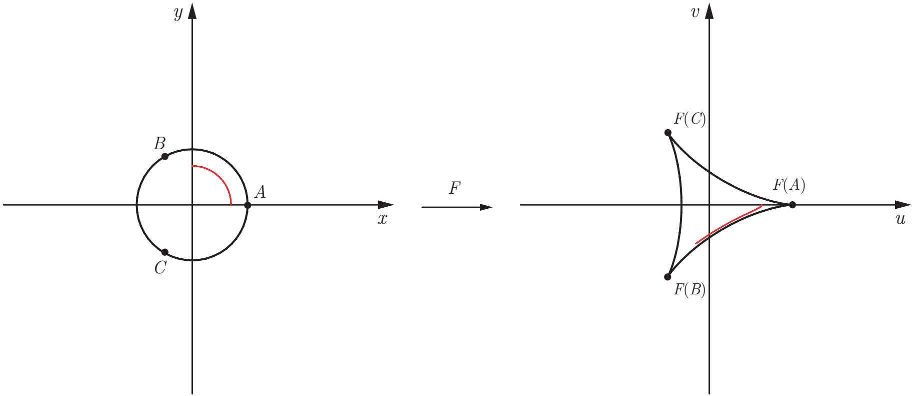

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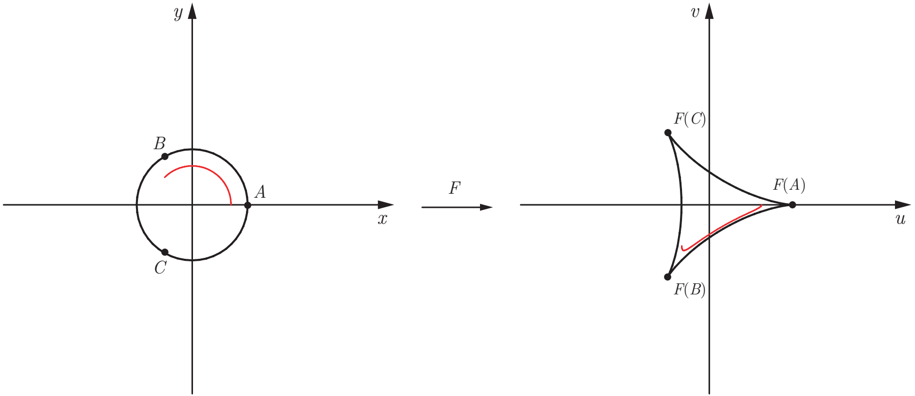

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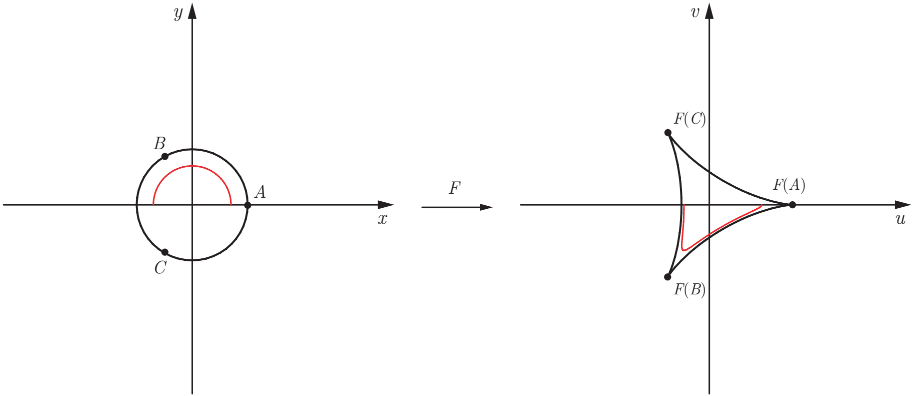

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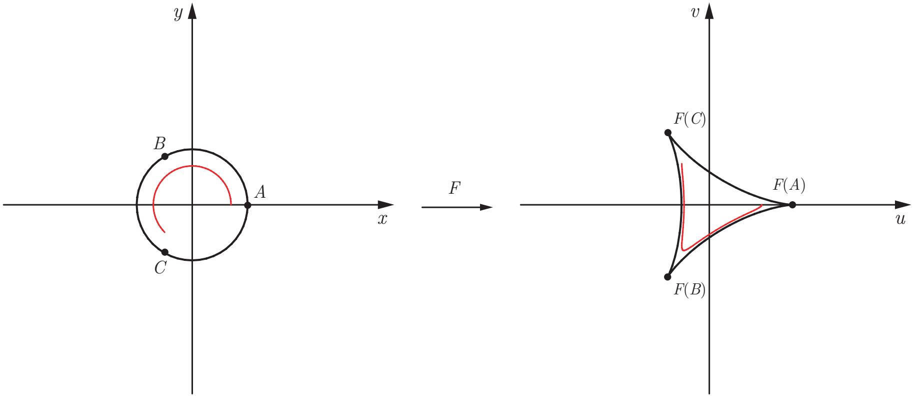

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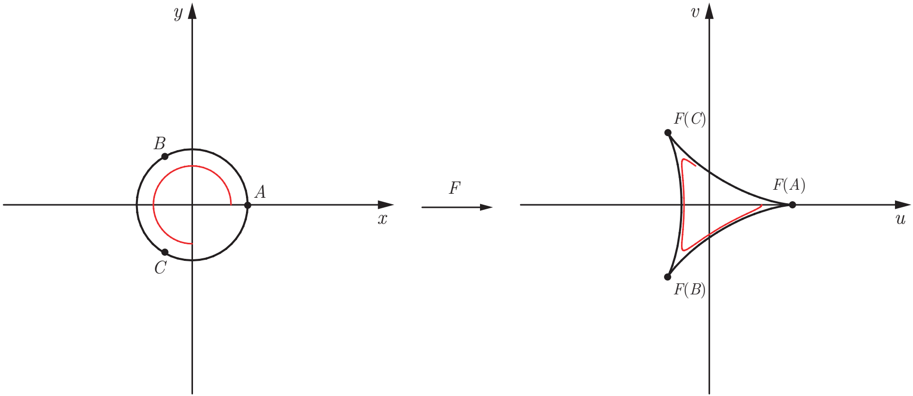

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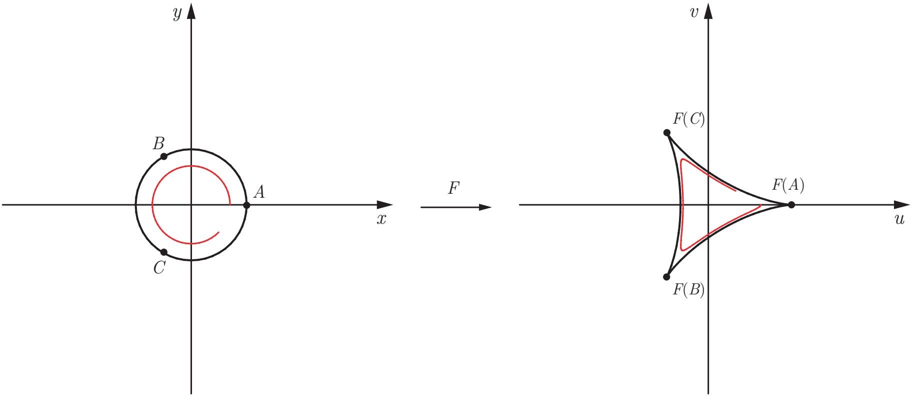

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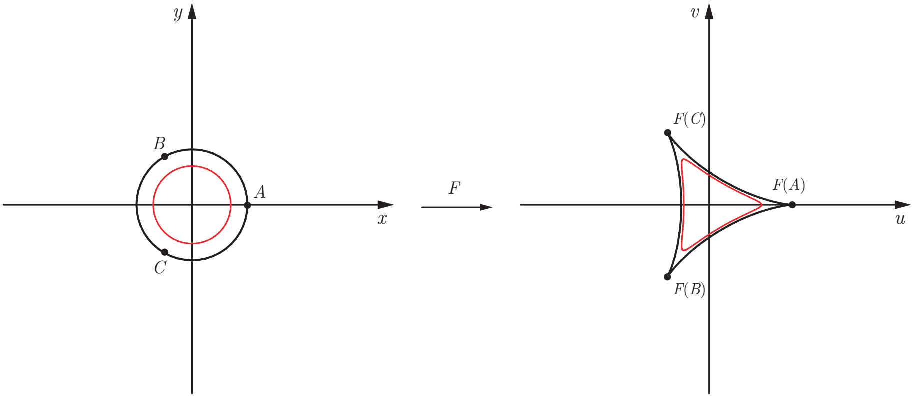

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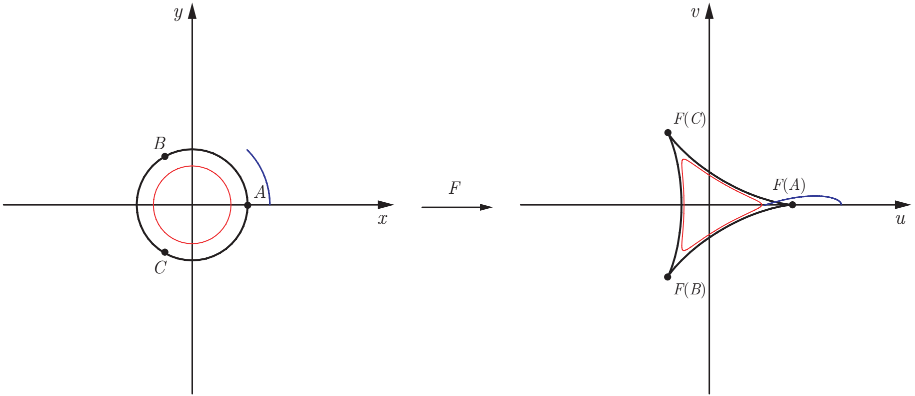

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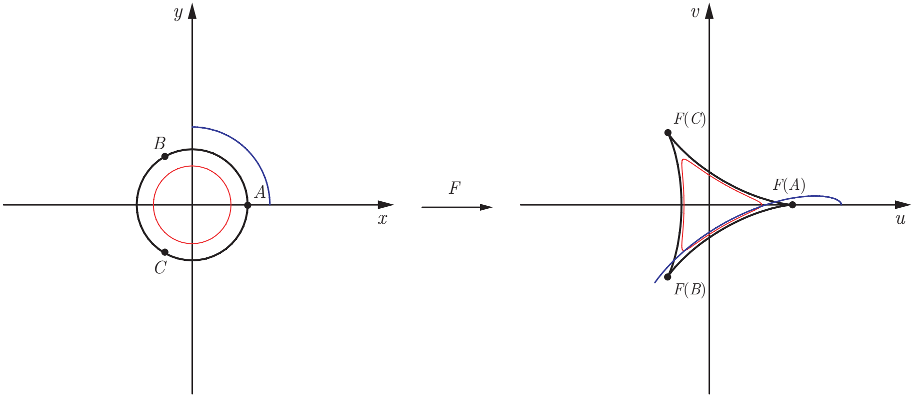

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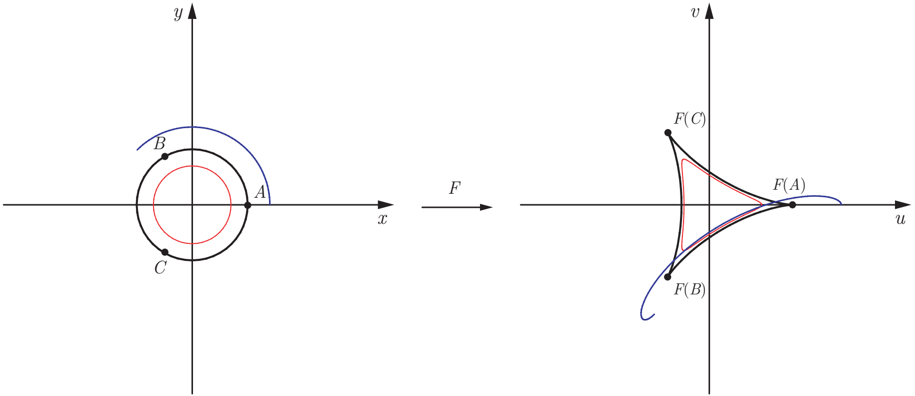

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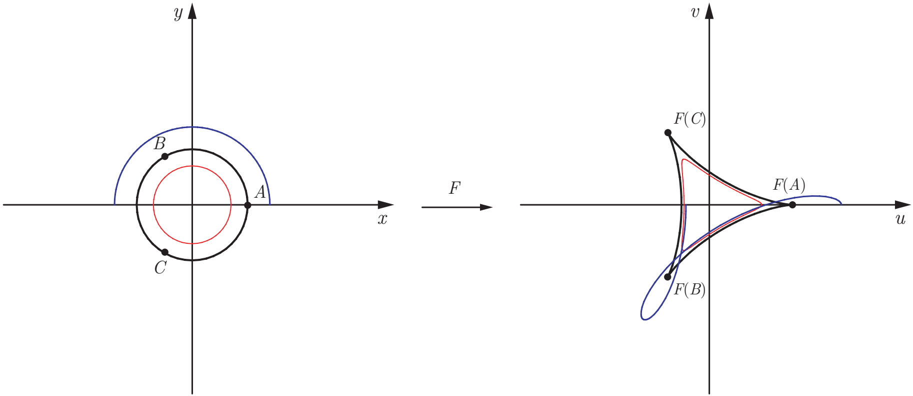

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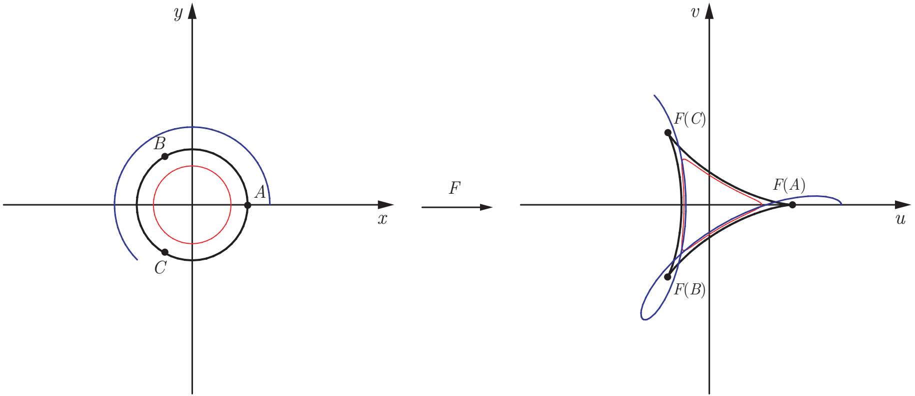

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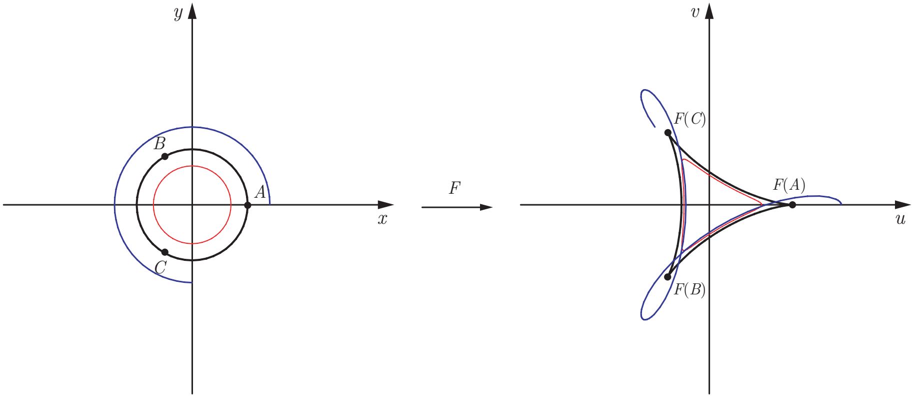

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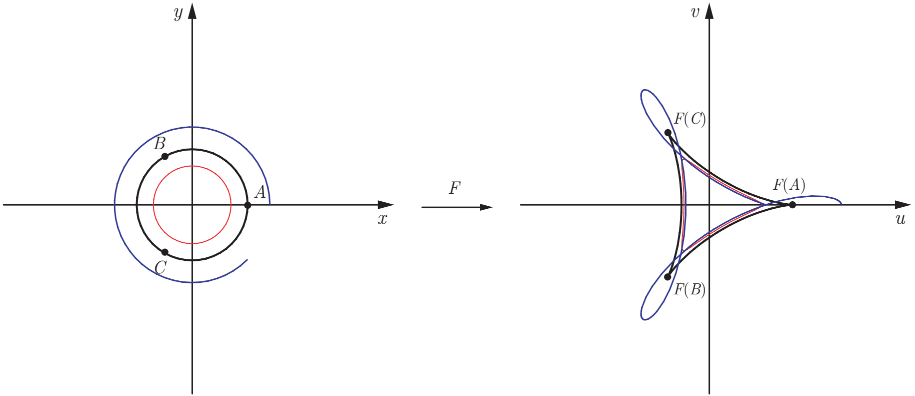

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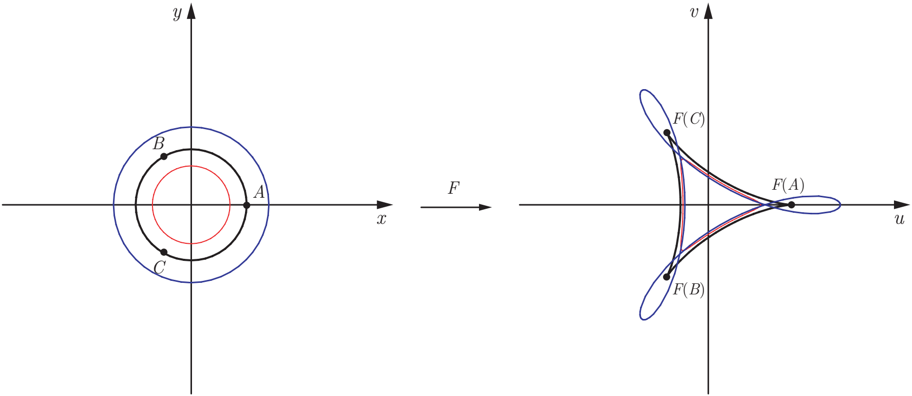
$$F(x, y) = (x^2 - y^2 + x, 2\,xy - y) \qquad \widetilde{F}(z) = z^2 + \overline{z}$$



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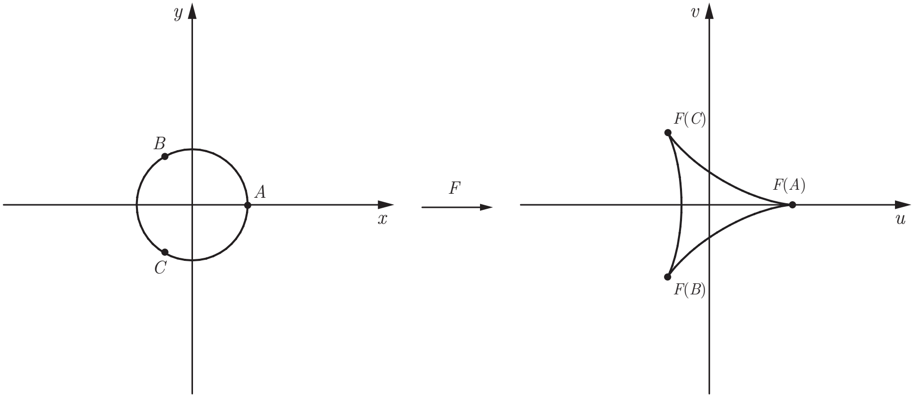

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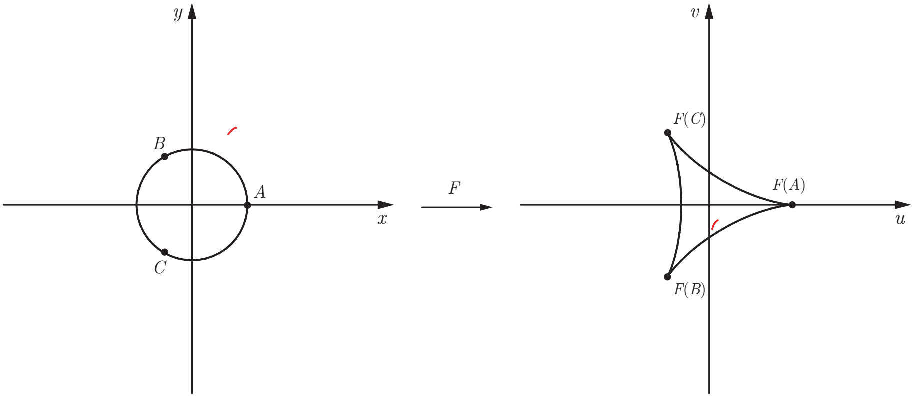

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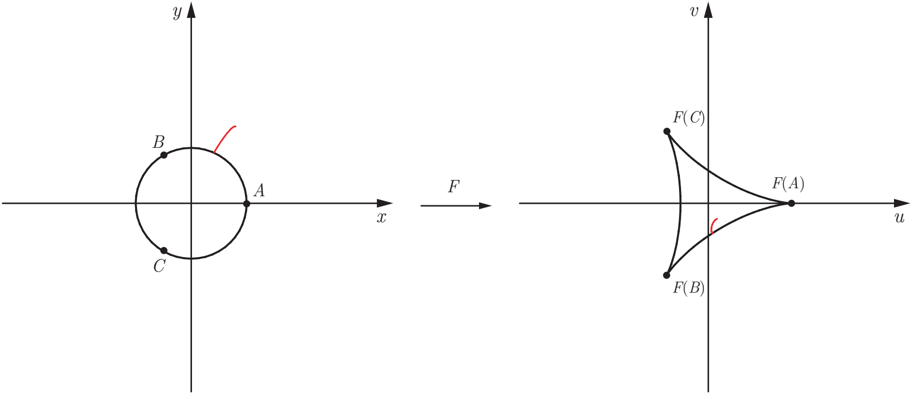

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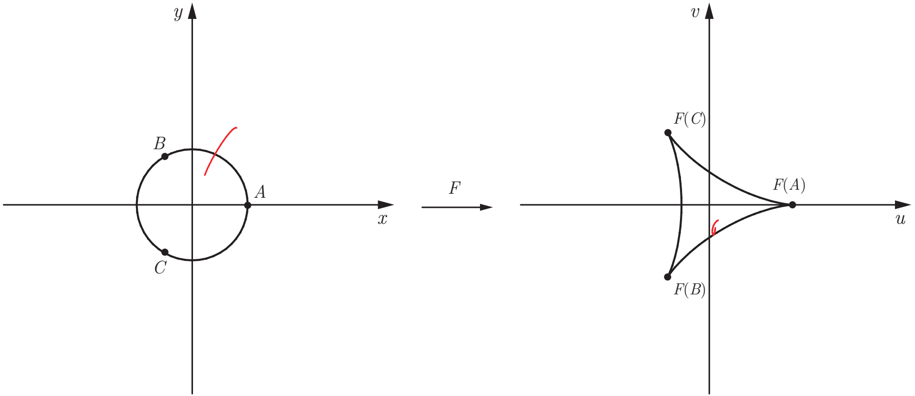

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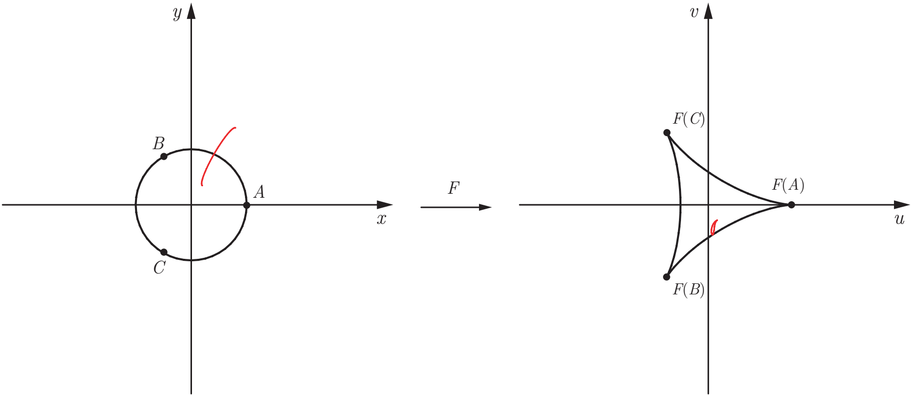

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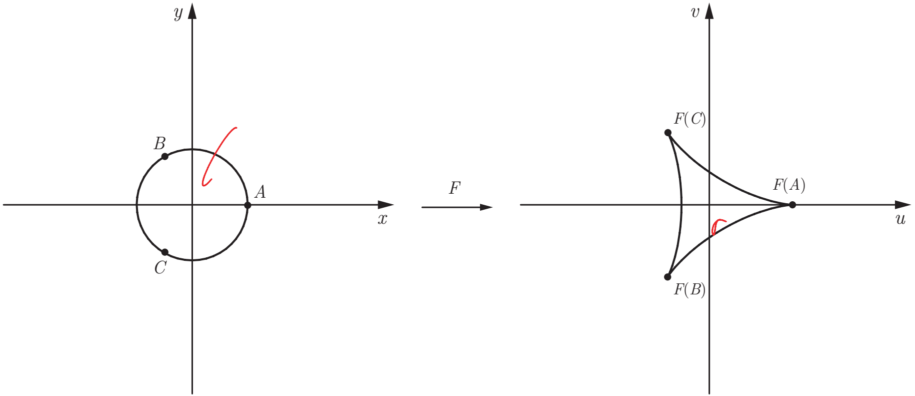

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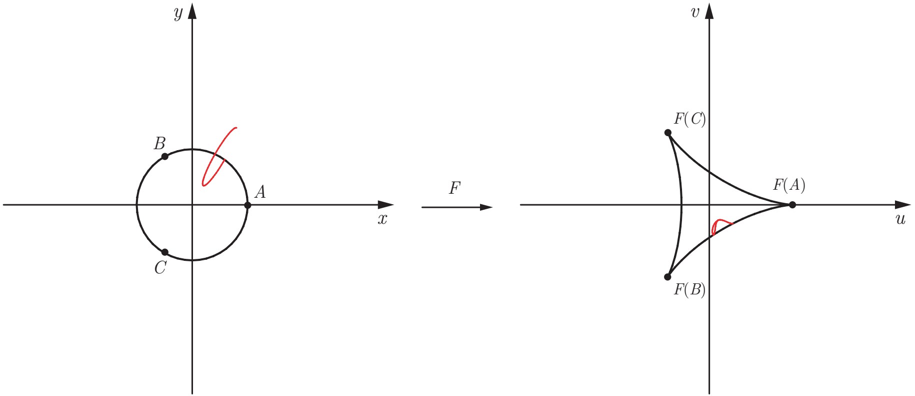

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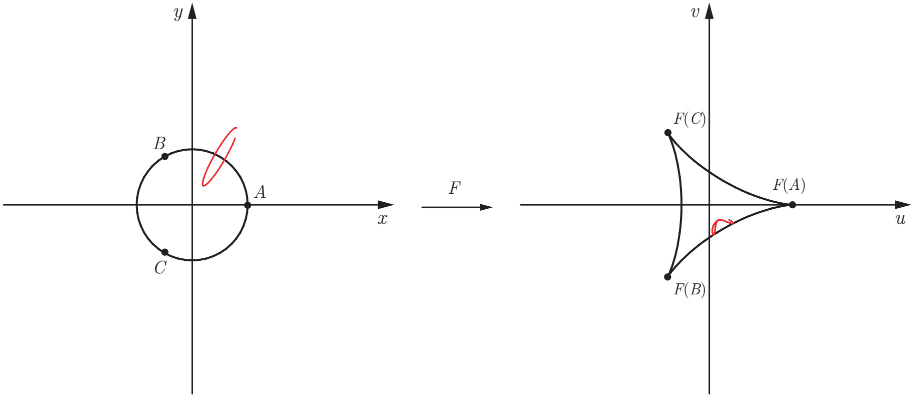

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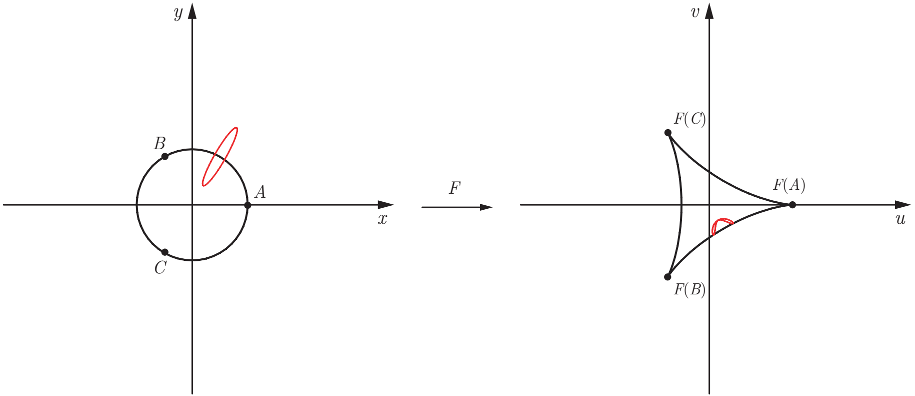

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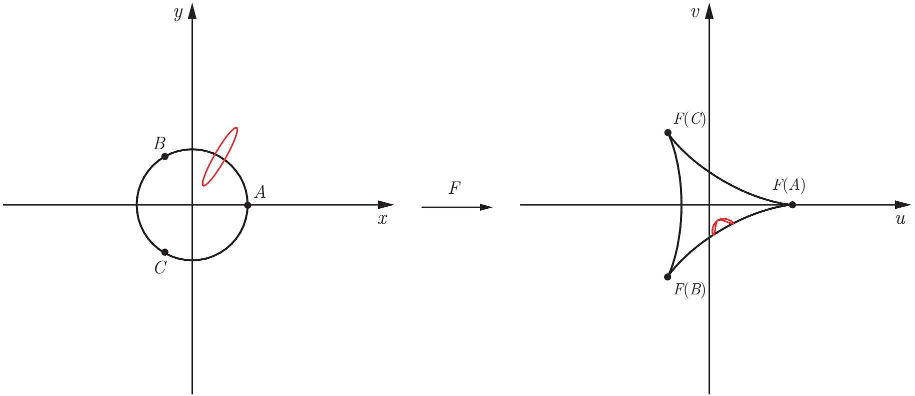

Forma normal: $(x, v) \mapsto (x^2, v)$.

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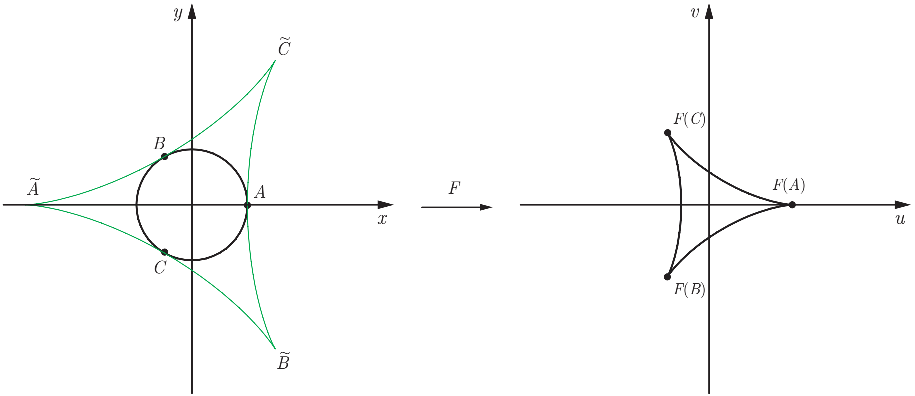

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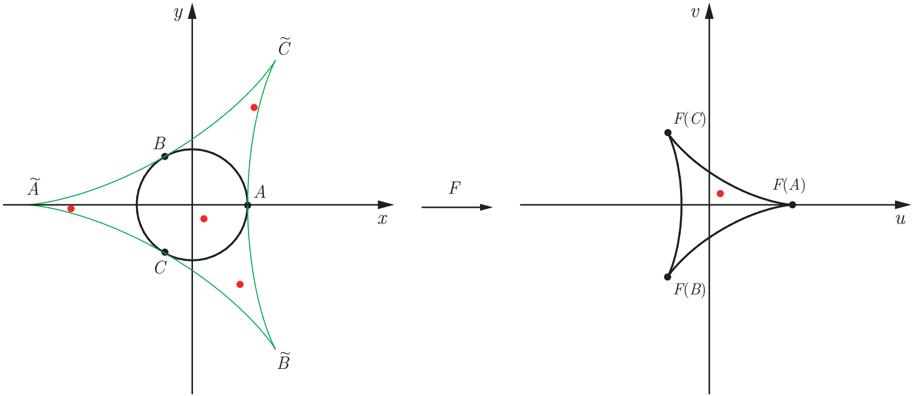

A equação $(x^2 - y^2 + x, 2\,xy - y) = (0.1, 0.1)$ tem 4 soluções.

$z \mapsto z^2 + \overline{z}$: cúspides

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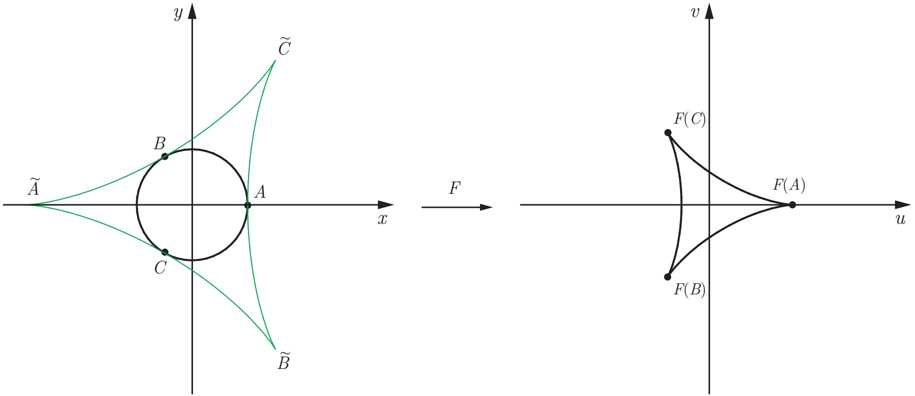

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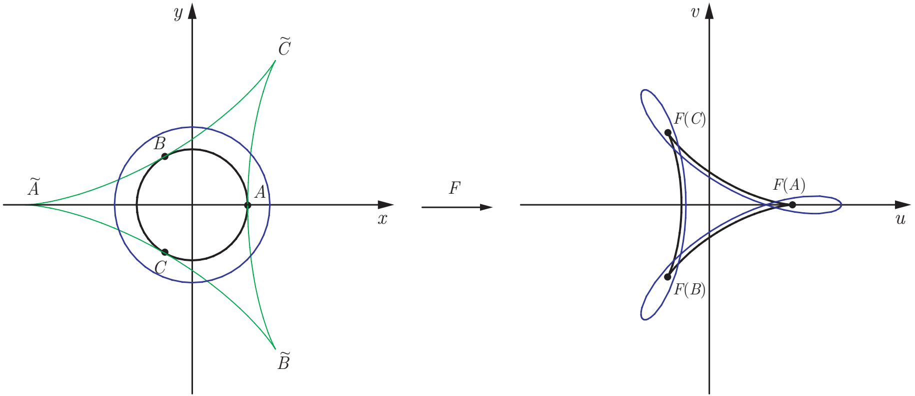

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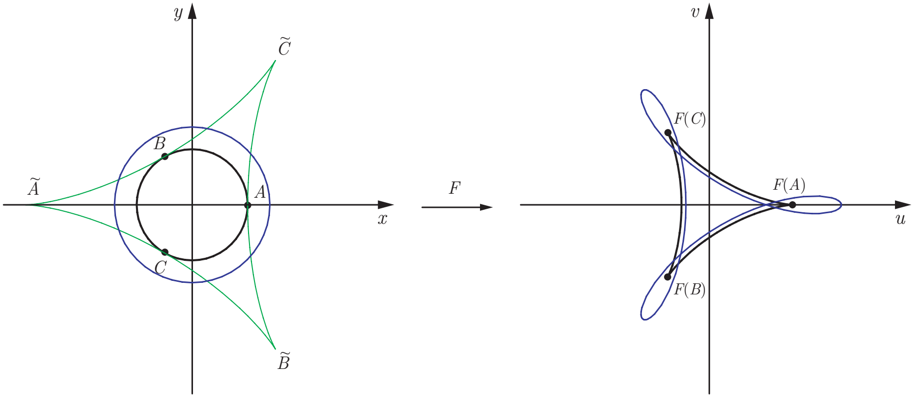

Forma normal da cúspide: $(x, y, v) \mapsto (x, y^3 - xy, v)$.

Em geral, a grande pergunta é . . .

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Dada $F\colon \mathbb{R}^2 \to \mathbb{R}^2$ suave genérica e seu comportamento em ∞, como identificar C, $F(C)$?

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Agora, resolver $F(x, y) = (a, b)$ não é tão difícil!

$$\begin{array}{rccc} F: & C^1(\mathbb{S}^1) & \to & C^1(\mathbb{S}^0) \\ & u & \mapsto & u' + u^2 \end{array}$$ é uma dobra global.

Além do plano: vendo como um matemático

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Assim, $u'(t) + u^2(t) = g(t)$, $u(0) = u(1)$, tem 0, 1 ou 2 soluções.

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E muito mais.

O programa 2×2 está disponível no endereço:

http://www.mat.puc-rio.br/∼nicolau/2x2/2x2.html